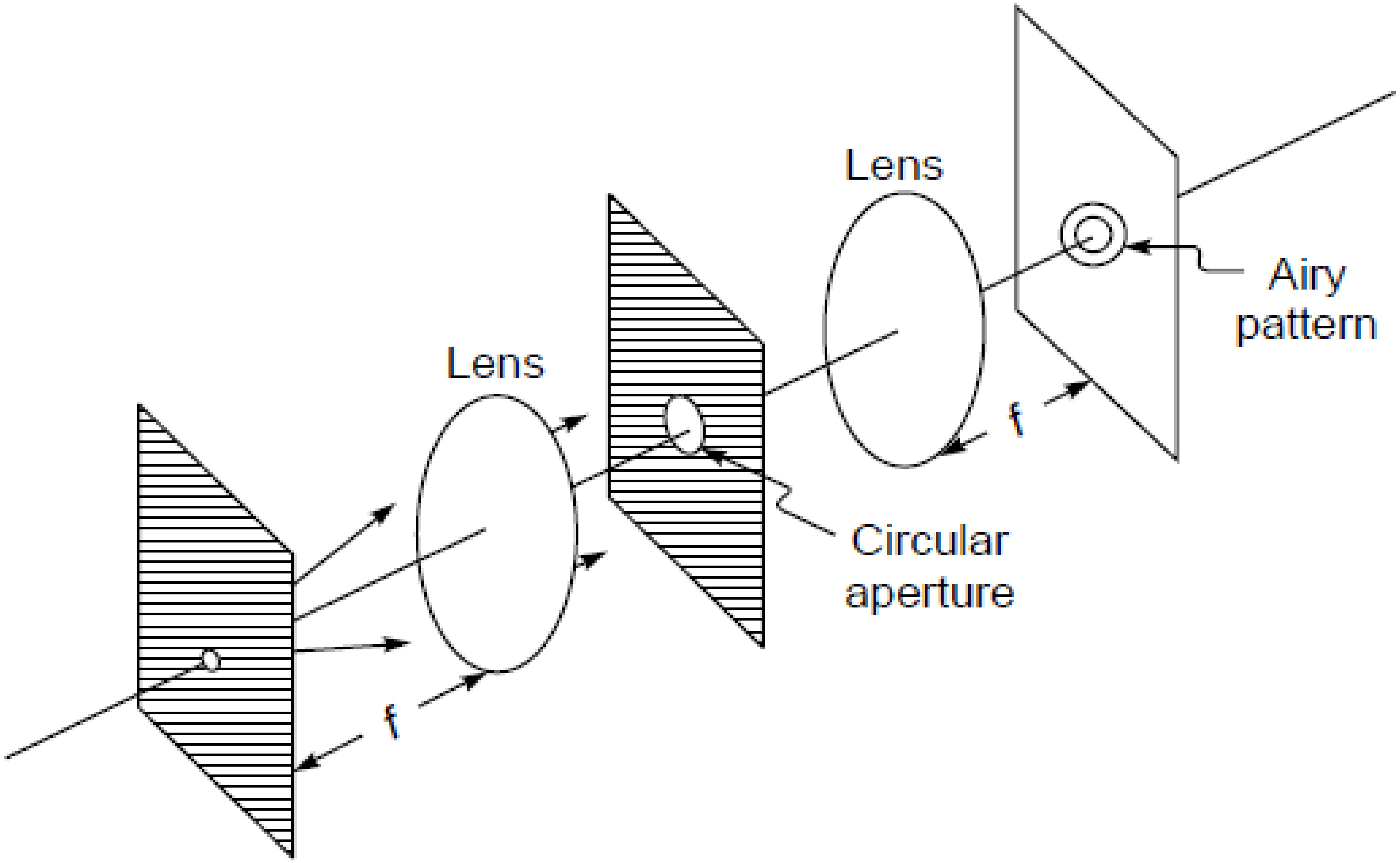
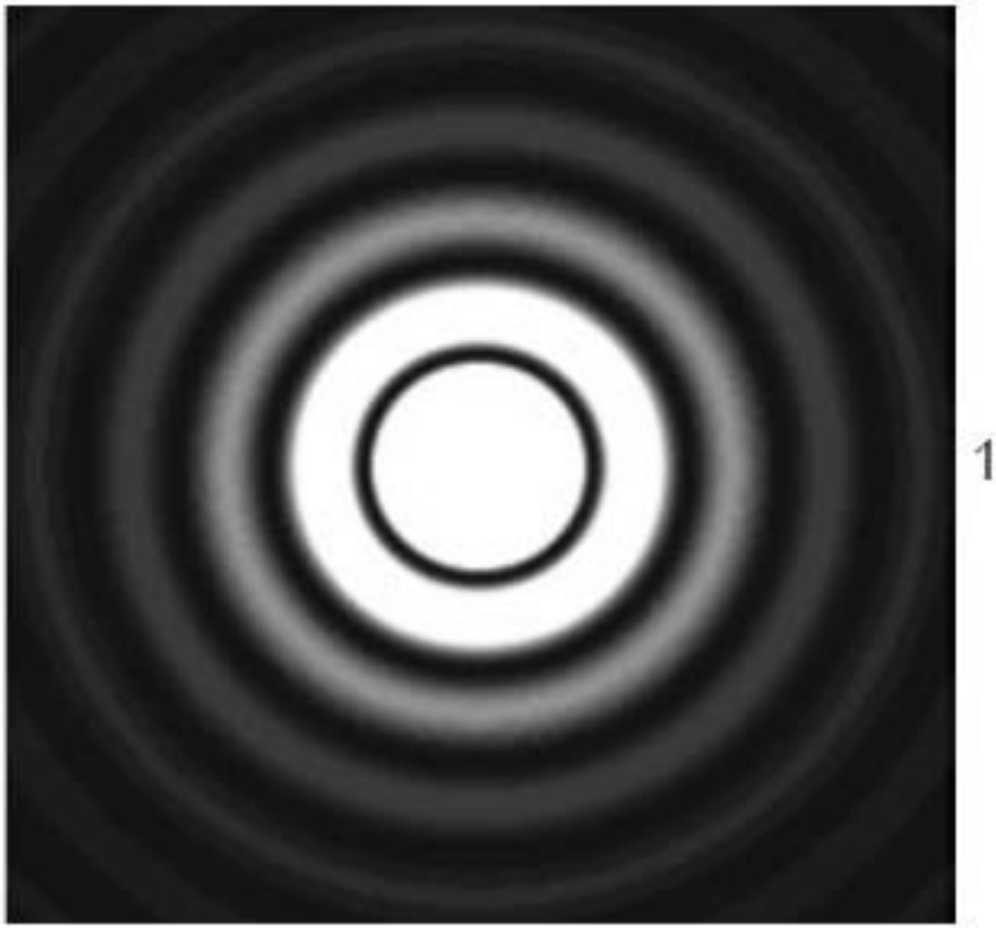
Diffraction By Circular Aperture


Lens
Lens
Airy
pattern
f
f
Circular
aperture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- A plane wave is incident normally on the circular aperture.
- Lens whose diameter is much larger than that of the aperture and the Fraunhofer diffraction pattern is observed on the focal plane of the lens.
- It is because of the rotational symmetry of the system
- The diffraction pattern will consist of concentric dark and bright rings
- This diffraction pattern is known as the Airy pattern.

- Direction of first minimum

$$\theta \approx 1.22\frac{\lambda}{d}$$

θ = angle subtended at the center of aperture by radius of the first dark ring.

f = focal length of lens

r = radius of first dark ring

$$r = \theta f = \frac{1.22\lambda f}{d}$$

Intensity distribution

$$I = I_0 \left[\frac{2J_1(v)}{v} \right]^2$$

$$v = \frac{2\pi}{\lambda} a \sin\theta$$

a = radius of circular aperture
λ = wavelength of light used
θ = angle of diffraction
$\theta = 0 \rightarrow I = I_o$
$J_1(\upsilon)$ = Bessel function of first order

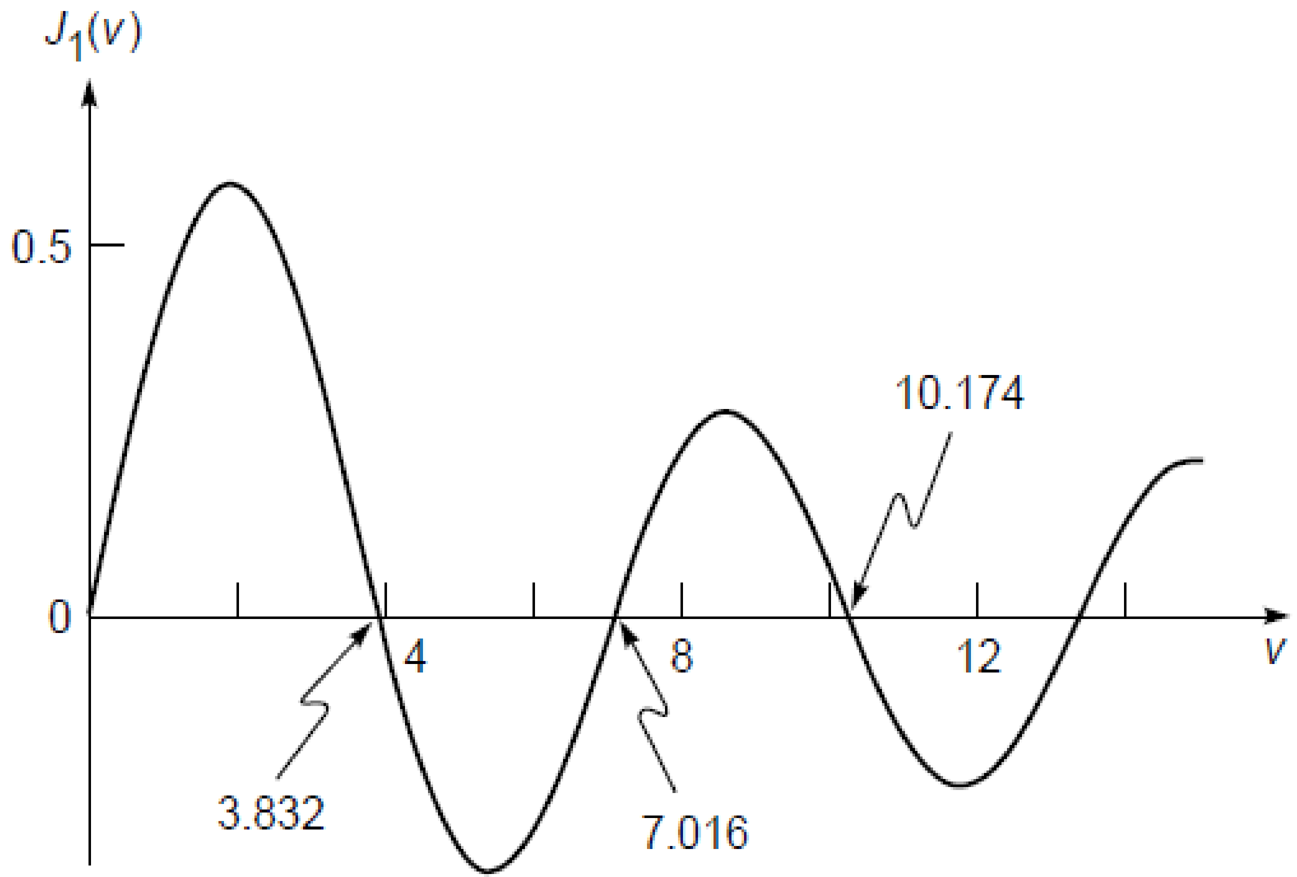
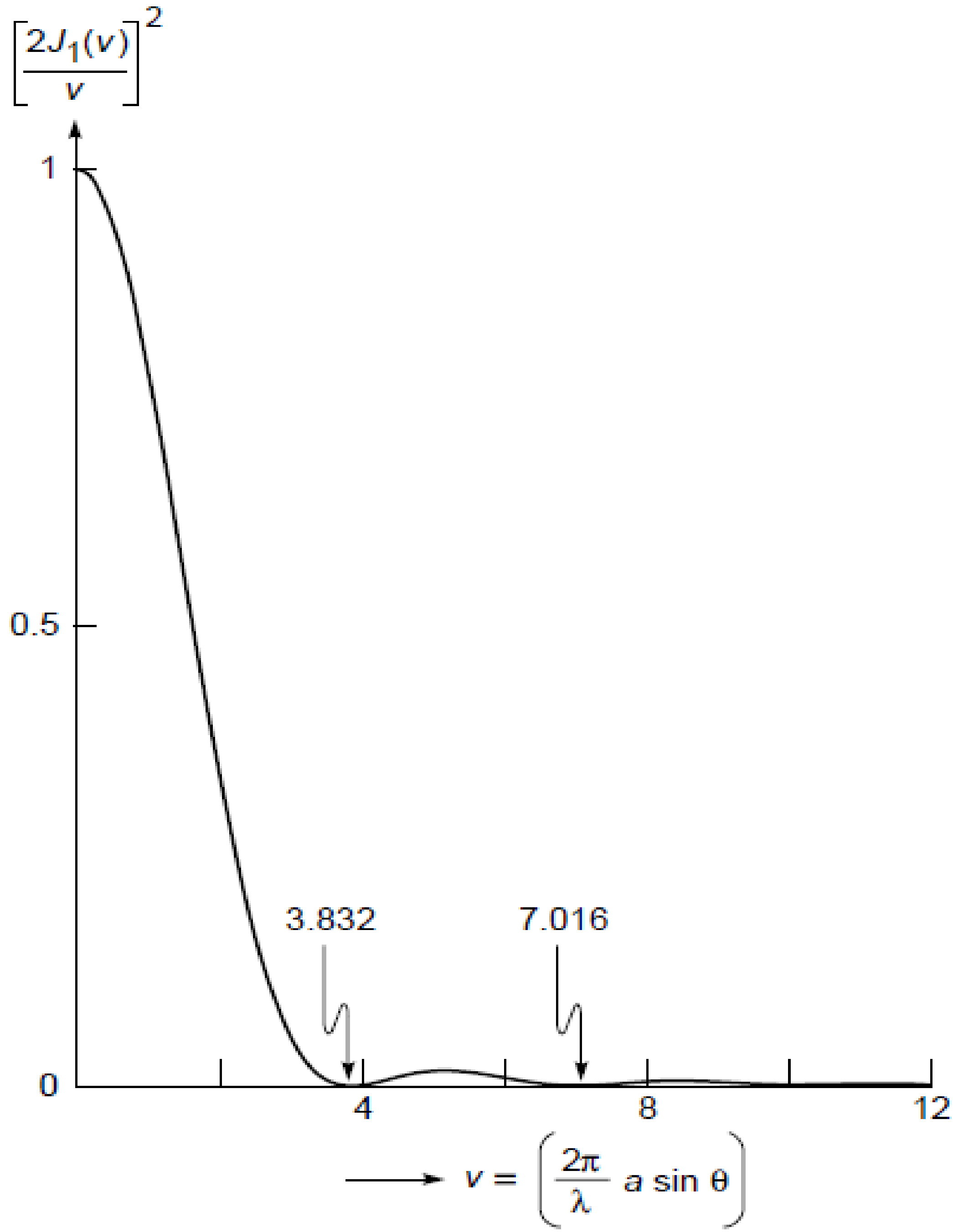


Fig. 18.10 The variation of $J_1(v)$ with v.



$\left[\frac{2J_1(v)}{v}\right]^2$
1
0.5
0
3.832
7.016
4
8
12
$v = \left(\frac{2\pi}{\lambda} a \sin\theta\right)$

- At central maxima $\theta = 0$, $I = I_o$
- $J_1(u) = 0$ at

u = 3.8317, 7.0156, 10.1735, 13.3237, 16.4706,……

corresponds to dark rings.

$$\nu = \frac{2\pi}{\lambda} a \sin\theta = 3.8317, 7.0156, ...$$

$$\frac{2\pi}{\lambda} a \sin\theta = 3.8317, \ first\_dark\_ring$$

$$\sin\theta = \frac{3.8317\lambda}{2\pi a} = \frac{1.22\lambda}{2a} = \frac{1.22\lambda}{d}$$

$$\theta \approx \frac{1.22\lambda}{d}$$

- For second dark ring

$$\frac{2\pi}{\lambda} a \sin\theta = 7.0156, \mathrm{Second}\_dark\_ring$$

$$\sin\theta = \frac{7.0156\lambda}{2\pi a} = \frac{2.234\lambda}{2a} = \frac{2.234\lambda}{d}$$

$$\theta \approx \frac{2.234\lambda}{d}$$

- Angular spread of the beam

$$\Delta\theta \sim \frac{1}{2}\frac{1.22\lambda}{d} \approx \frac{\lambda}{d}$$